Vibration Testing of Small Satellites

This series of papers provides a tutorial along with guidelines and recommendations for vibration testing of small satellites. Our aim with these papers is to help you (a) ensure the test meets its objectives in demonstrating flight worthiness and (b) avoid test failures, whether associated with a design deficiency or with excessive loading during test. Addressed are sine-burst testing, random vibration testing, and low-level diagnostic sine sweeps. Although much of the guidance provided in this series applies to CubeSats, the series is primarily aimed at satellites in the 50 – 500 lb (23 – 230 kg) range. Most of the guidance applies to larger satellites as well if they will be tested on a shaker.

The plan is for this series to include seven parts, each of which will be released when completed:

1. Introduction to Vibration Testing (originally released April 11, 2014; revised since then)
2. Test Configuration, Fixtures, and Instrumentation (originally released April 11, 2014; revised since then)
3. Low-level Sine-Sweep Testing (originally released May 13, 2015; revised since then)
4. Sine-Burst Testing (originally released April 28, 2017; revised since then)
5. Random Vibration Testing (originally released April 7, 2016; last revised July 24, 2017)
6. Notching and Force Limiting (originally released May 13, 2015; revised since then)
7. Designing a Small Satellite to Pass the Vibration Test (yet to be released)

The most recent versions of these papers are available for free download at

http://instarengineering.com/vibration_testing_of_small_satellites.html.

Part 5: Random Vibration Testing

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In a random vibration test, the shaker imparts vibration at multiple frequencies simultaneously, with randomly varying acceleration at each frequency. Such tests attempt to simulate any of a variety of environments, such as vibration during ground transportation, the effect of ocean waves, and vibration caused by acoustic or aerodynamic pressure fluctuation during launch of spaceflight hardware. Figure 5-1 shows an example of random vibration as an acceleration time history.

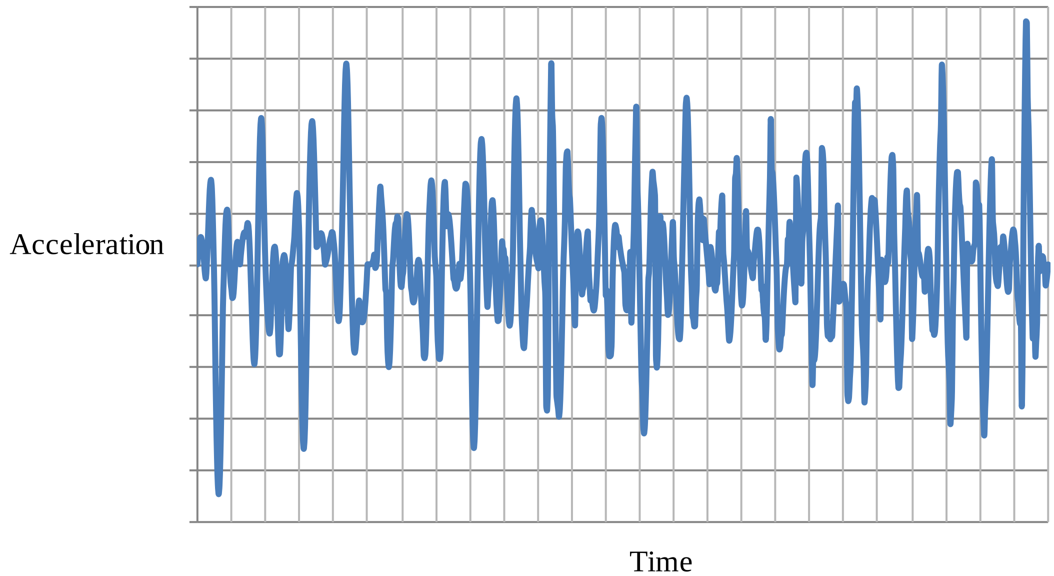

Fig. 5-1. Example of Random Vibration.

Some amount of random vibration is present throughout a launch vehicle (LV) for the full duration that engines are firing. However, levels seen by LV payloads are typically significant only during liftoff, when acoustic waves reflect off of surrounding structures, and during the transonic phase of ascent. Total duration of high levels of random vibration is normally less than 15 seconds.

In the space industry, a random vibration test typically has frequency content from 20 to 2000 Hz. The test should be (and typically is) controlled such that the statistical distribution of acceleration at any point in time is Gaussian (Ref. 1, NASA-STD-7001A, Sec. 4.3.2.4).

Acceleration Spectral Density

By definition, random acceleration is not predictable at any point in time, so we don't use a time history of acceleration, such as the one shown in Fig. 5-1, to define a random vibration environment. Instead, we define the environment by its frequency content. We do this with ***acceleration spectral density***[1] ***(ASD)*** vs. frequency over a frequency range. Starting with a time history, which we can measure over a sample time period, we break the full frequency range into bands and calculate the ASD for each band, including only the frequency content of acceleration within that band. For a given frequency band, with *f* as the center, the ASD, W(*f*), is the mean-square acceleration within that frequency band divided by the bandwidth, Δ*f*.

$$W(f) = \frac{1}{\Delta f}\left[\frac{1}{T}\int_0^T a^2 dt\right] \quad (5.1)$$

[1] More generally, this term is referred to as ***power spectral density (PSD)*** for processing of any random signal. This method of processing a random signal was first used in electrical engineering. The term "power" comes from the power dissipated in a resistor from a randomly varying electrical current.

where a is acceleration within the frequency band, and T is the duration of the random vibration. Typical ASD units are g^2/Hz. Following is a simplified example of ASD calculation.

Example 5-1: Calculating ASD from a Time History

Suppose we put an accelerometer on a vibrating machine and, over 0.2 sec, record the time history shown in Fig. 5-2. This random signal is actually the combination of the three sinusoidal functions shown in Fig. 5-3, each with a different frequency and randomly varying amplitude. (In this simplified example, the three sine functions were generated first and then combined to get the time history in Fig. 5-2. In practice, we typically start with a measured random signal, such as the one in Fig. 5-2, and then decompose it into its frequency content with Fourier analysis. ASD is calculated using the fast Fourier transform.)

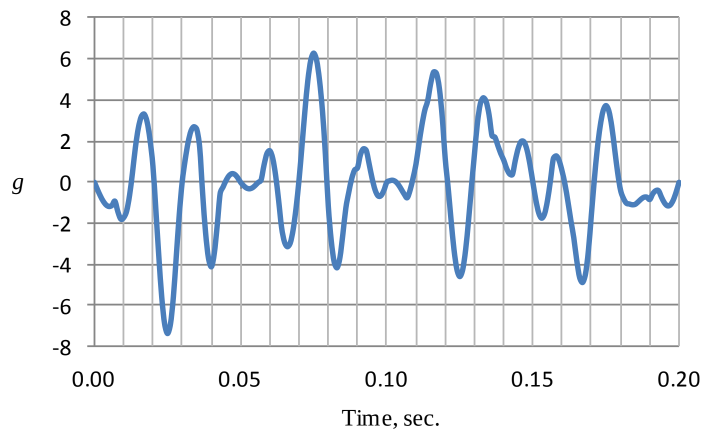

Fig. 5-2. Hypothetical Time History of Random Acceleration.

When processing a time history into an ASD, we break the full frequency range into bands and include the acceleration only within a given band. In this example, starting at 20 Hz and using a frequency bandwidth of 20 Hz, we can calculate three ASD values for frequency bands centered at 30 Hz, 50 Hz, and 70 Hz, corresponding to the three acceleration time histories in Fig. 5-3.

In trying to characterize the acceleration within a given frequency band, calculating the average value is pointless because the average is zero once any applicable steady-state acceleration is removed from the data. If we square the random acceleration, though, all values are positive, and the average has meaning. Figure 5-4 shows a plot of 50-Hz acceleration squared compared with the original 50-Hz acceleration time history.

The mean square acceleration within a given frequency band is the area under the time-history plot of squared acceleration divided by the duration of the time history. If we divide that value by the frequency bandwidth (20 Hz in this example), per Eq. 5.1, we get the ASD in g^2/Hz.

Table 5-1 shows a spreadsheet used to numerically integrate each of the frequency time histories for our example.

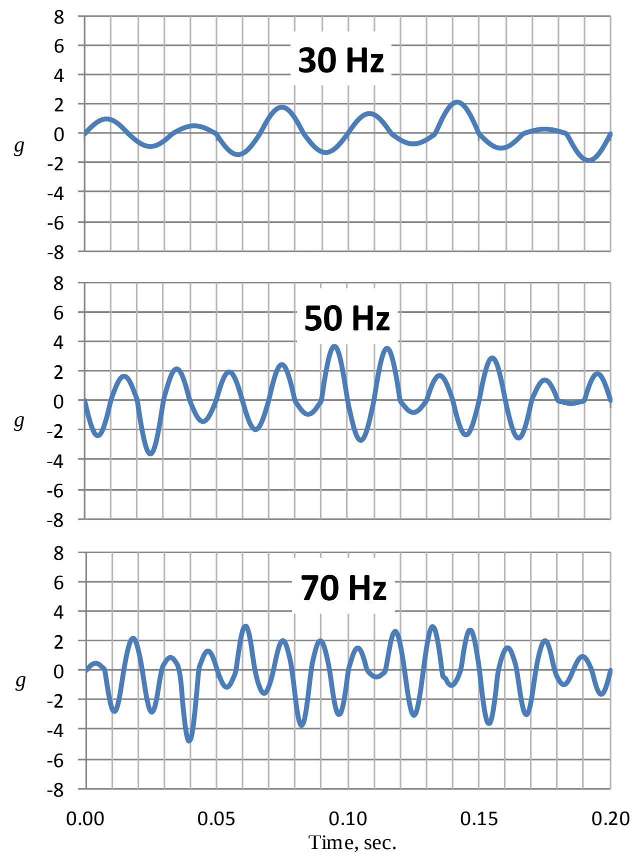

Fig. 5-3. Frequency Content of the Random Acceleration in Fig. 5-2. When added, these three functions combine to equal the time history shown in Fig. 5-2.

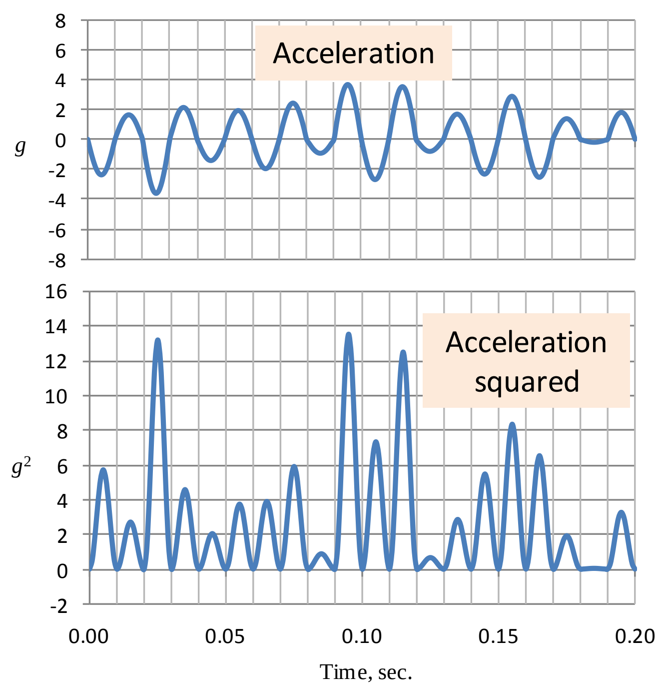

Fig. 5-4. Time Histories of 50-Hz Acceleration and Acceleration Squared.

Table 5-1. Calculation of ASDs within Three Frequency Bands for the Example in Fig. 5-2. The overall or combined acceleration, $a = a_1 + a_2 + a_3$ at each time point.

Overall acceleration (g) — 30-Hz acceleration — 50-Hz acceleration — 70-Hz acceleration

t (sec)	a	a_1	a_1^2	a_1^2 dt	a_2	a_2^2	a_2^2 dt	a_3	a_3^2	a_3^2 dt
0.000	0.00	0.00	0.00	0.00000	0.00	0.00	0.00000	0.00	0.00	0.00000
0.001	-0.34	0.19	0.04	0.00004	-0.74	0.54	0.00054	0.21	0.04	0.00004
0.002	-0.66	0.37	0.14	0.00014	-1.40	1.97	0.00197	0.37	0.14	0.00014
0.003	-0.92	0.54	0.30	0.00030	-1.93	3.73	0.00373	0.47	0.22	0.00022
…	…	…	…	…	…	…	…	…	…	…
…	…	…	…	…	…	…	…	…	…	…
0.197	-1.10	-0.98	0.95	0.00095	1.46	2.13	0.00213	-1.59	2.51	0.00251
0.198	-0.87	-0.67	0.45	0.00045	1.06	1.13	0.00113	-1.26	1.59	0.00159
0.199	-0.48	-0.34	0.12	0.00012	0.56	0.31	0.00031	-0.70	0.49	0.00049
0.200	0.00	0.00	0.00	0.00000	0.00	0.00	0.00000	0.00	0.00	0.00000
Sum (area under the curve), g^2-sec				0.17108			0.52371			0.58094
Mean square (sum ÷ 0.2 sec), g^2				0.86			2.62			2.90
ASD (mean square ÷ 20 Hz), g^2/Hz				0.043			0.131			0.145

Over the full frequency range of 20 – 80 Hz, the mean square acceleration is equal to the sum of the individual mean-square values: 0.86 + 2.62 + 2.90 = 6.38 g^2. The square root of this value, 2.53 g, is the root-mean-square acceleration.

Calculation and Significance of the Root-Mean-Square Acceleration

The ***root-mean-square (RMS)*** value is the standard deviation (σ) of acceleration as it varies randomly over time. We often consider the 3σ value (3 times the RMS) to be the maximum level achieved over the duration of the event, but this is not actually the case. In the above example, the 3σ acceleration is 3(2.53) = 7.59 g. From the original time history in Fig. 5-2, the actual absolute peak acceleration is 7.35 g. This peak is a 7.35/2.53 = 2.91σ value—a little less than 3σ. This peak occurred in just 0.2 seconds duration of the environment. If we repeat the experiment, once again recording and processing random acceleration for this environment over 0.2 sec, we would expect to see a different peak value. If we extend the duration to, say, 60 sec, which is the typical duration of an acceptance random vibration test for flight hardware (see below), we almost assuredly would see a peak higher than 3σ.

For a linear system[2], it's not uncommon to see peak response accelerations of 4.5 or 5σ in a 60-second test.[3] (See Sec. 12.2.2 of Ref. 3 for a discussion of probability associated with random vibration.) Still, many engineers use the 3σ prediction as the ***limit*** (maximum expected) ***load*** for strength analysis. This approach may be justified for assessment of ductile failure modes; yielding, which causes deviation from linearity, essentially absorbs energy and often prevents occurrence of 4.5 and 5σ peak responses. For the

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[2] In this context, a ***linear system*** is a structure whose displacement is proportional to the applied load, which means its natural frequencies are constant regardless of the acceleration or inertia load level, and whose damping is also constant regardless of acceleration or inertia load. No actual structural assembly is perfectly linear, but many are nearly so up to the point at which the materials are stressed to the proportional limit, and linearity is a common assumption in analysis.

[3] Many shakers offer 3σ clipping, which keeps the amplitude of the control signal from exceeding +/-3 standard deviations. Such clipping, however, does not keep acceleration of the shaker or the test article from exceeding 3σ. See Ref. 2 for details on clipping.

same reason, we often use the 3σ load as a basis for force limiting (see Part 6 in this series of papers). When a ductile material ruptures as a result of random vibration, it's usually a fatigue failure. With structures made of brittle materials, 4.5 or 5σ may be more appropriate for strength analysis and force limiting.

We customarily plot ASD on a log-log scale. Figure 5-5 shows such a plot for the Example 5-1.

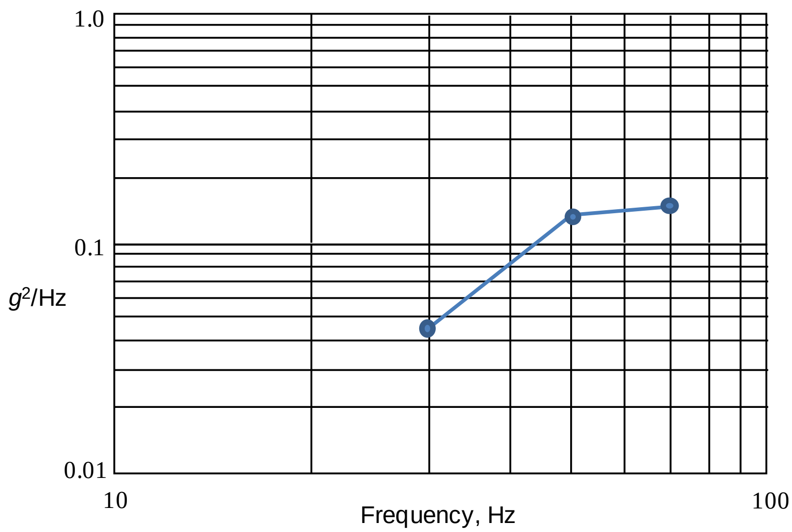
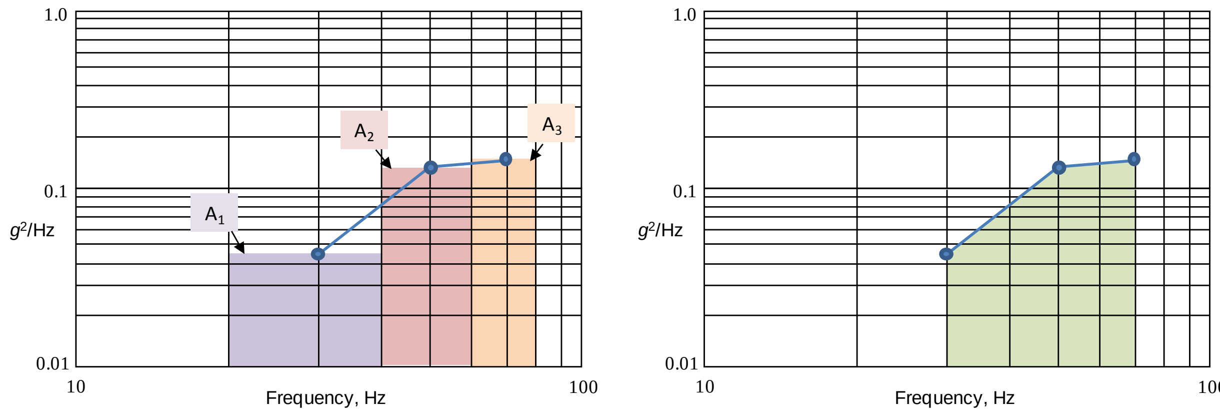


Fig. 5-5. ASD Plot Corresponding to Table 5-1 from Example 5-1.

The RMS value is approximately equal to the square root of the area under the ASD curve; we use the word "approximately" only because of the finite bandwidth for data processing. For the above example, the approximation is not very close because of the low data resolution (large frequency bandwidth) used to process the raw data into ASD and the fact that the example only had three discrete frequencies of content. ***Data resolution*** indicates the number of calculated ASD values (and number of frequency bands) over the applicable frequency range, e.g., 20 – 2000 Hz. Each frequency band is often referred to as a ***line of control***. Higher resolution means smaller bandwidth and more lines of control. Additional discussion of data resolution appears later in this paper.

Figure 5-6 shows (a) how the true RMS value is calculated in Example 5-1, compared with (b) the approximation from the square root of the area under the ASD plot.



a. The RMS is equal to the square root of the sum of the three areas shown above, where A_1 = 20 Hz (0.043 g^2/Hz) = 0.86 g^2, etc.

b. Because of the low resolution, the area under the plotted ASD is not a very close approximation of the total area shown at left.

Fig. 5-6. Calculating the RMS Value from the Example 5-1 ASD.

Decibels

When an ASD is increased or decreased, we refer to that level change in ***decibels (dB)***. At a given frequency, the decibel change, N_{dB}, when going from an initial ASD, W_1, to a final ASD, W_2, is defined as

$$N_{\mathrm{dB}} = 10\left[\log_{10}\left(\frac{W_2}{W_1}\right)\right] \tag{5.2}$$

Conversely, the ratio by which an ASD increases for a given decibel change is approximated as

$$\frac{W_2}{W_1} \cong 2^{\left(N_{dB}/3\right)} \tag{5.3}$$

For example, for a 6 dB increase, the g^2/Hz goes up approximately by a factor of four, which means the acceleration doubles (square root of four). Thus, increasing the entire environment by 6 dB means that, for a linear system, the loads and stresses in the test article double. A 3-dB increase means the acceleration increases by approximately 1.4 (square root of 2).

Deriving a Test Environment

Figure 5-7 shows a hypothetical example of what a plot of ASD might look like for random vibration containing frequency content from 20 – 2000 Hz.

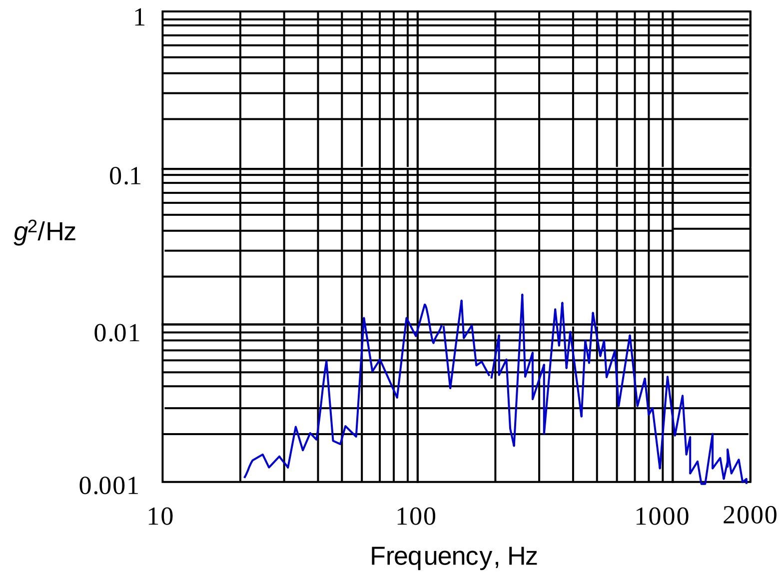


Fig. 5-7. Hypothetical Example of ASD Processed from 20 – 2000 Hz.

Let's assume the plot shown in Fig. 5-7 is the result of processing a time history of random acceleration measured during launch at the mounting interface for a small satellite. To establish a suitable environment for testing a similar satellite that will be mounted in that location during launch, the first thing we typically do is envelop the data with a smooth curve, as shown in Fig. 5-8. There are two reasons for doing this: (1) during the next launch, the peaks and valleys in the ASD won't all be at the same frequencies, and (2) we want a smoother ASD plot (function, actually) to enable better control during the test.

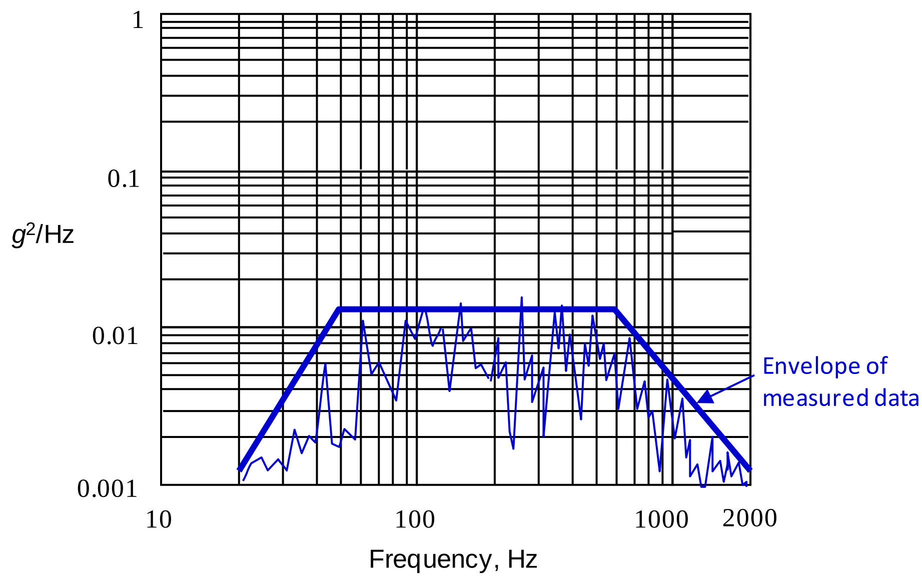


Fig. 5-8. Envelope for Use in Test. Note that the envelope does not fully encompass the measured data. Narrow-band peaks are commonly clipped, as was done here, because they have little associated energy (they don't contribute much to the total area under the curve) and thus relatively low damage potential.

The ASD envelope in Fig. 5-8 is not actually what we'll use for test. It's based on measured data from a single launch. We'll want to account for launch-to-launch variation.[4]

The ***maximum predicted environment (MPE)*** is an ASD envelope of the highest expected random vibration levels during the applicable event, such as launch, at a statistical basis of 95% probability with 50% statistical confidence (often referred to as a "95/50" environment). The statistical basis of 95% probability means that, to the best of our knowledge, there's only a 5% chance during the mission that the MPE will be exceeded at any given frequency; the 50% confidence means there's a 50-50 chance that the actual probability of a higher environment is greater than 5%. Such a statistical basis for MPE is the standard for military programs per SMC-S-016 (Ref. 4) and for NASA programs per NASA-STD-7001A.

SMC-S-016 and NASA-STD-7001A both require that, when measurements exist for only one launch, we must consider that data to represent a statistical mean. The MPE is then uniformly 4.9 dB higher than the mean per SMC-S-016 (5.0 dB higher than the mean per NASA-STD-7001A; the small difference here has to do with significant figures and rounding!).

Acceptance tests of flight hardware are done to MPE (or to the envelope of MPE and a specified minimum workmanship environment[5], if applicable). Figure 5-9 shows the MPE for our on-going example.

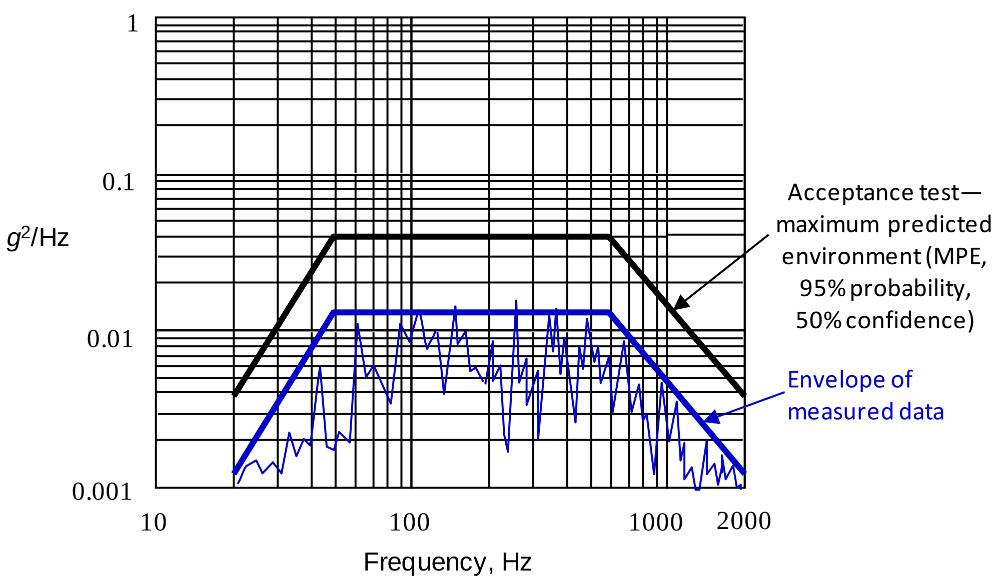


Fig. 5-9. Maximum Predicted Environment Derived from an Envelope of Data from a Single Launch. The MPE is either 4.9 dB or 5.0 dB (see text) above the measured data from a single flight. Acceptance testing is shown here as being to MPE, which is normally the case, but if there is a specified minimum workmanship environment that exceeds MPE within any frequency range, acceptance testing should be done to the envelope of MPE and minimum workmanship levels.

[4] There also may be an adjustment in either level or test duration if the high levels of random vibration during launch last for more than the typical 15 seconds. The adjustment is to achieve equivalent fatigue damage.

[5] A ***minimum workmanship environment*** is intended to ensure the hardware is adequately screened against workmanship-related defects. Even if the launch MPE is quite low within certain frequency bands, the hardware will experience random vibration during ground processing, handling, and transportation. SMC-S-016, NASA-STD-7001A, and GEVS (Ref. 5) specify minimum workmanship environments, either of which may apply to your program.

For the NASA protoflight qualification approach (see part 1 of this series of papers), NASA-STD-7001A requires testing the flight hardware to acceptance levels +3dB for 1 minute per axis. When a military program selects the protoqualification (often shortened to "protoqual") approach (also described in part 1 of this series), SMC-S-016 requires test environments that are 3 dB above acceptance levels, with a duration of 2 minutes per axis. For qualification testing of non-flight hardware, NASA-STD-7001A requires levels to be at acceptance levels +3dB, with a duration of at least two minutes per axis, whereas the most common option used on military programs, per SMC-S-016, is acceptance levels +6dB for 3 minutes per axis. Note that duration is an important criterion because most failures occurring during exposure to random vibration are material fatigue failures, which entail crack formation and growth under cyclic loading. This is especially true for ductile materials.

Table 5-1 summarizes the differences between military and NASA standards. The standard you should use depends on whether your small satellite is part of a military mission or a NASA mission. Most commercial programs adhere to NASA standards. Figure 5-10 shows Department of Defense (DOD) and NASA standard test levels for our example.

Table 5-1. Comparison of Government Standards for Random Vibration Testing of Spaceflight Hardware. Level is relative to maximum predicted environment, and duration is per axis. SMC-S-016 provides optional durations for qualification testing; the duration shown is the most common approach.

		SMC-S-016		NASA-STD-7001A	
Class of Test	Applicability	Level	Duration	Level	Duration
Qualification	Test-dedicated hardware	+6 dB	3 minutes	+3 dB	2 minutes
Protoqualification (a.k.a. protoqual) or Protoflight	First-built flight article in absence of a qualification test	+3 dB	2 minutes	+3 dB	1 minute
Acceptance	Each flight article other than a protoflight or protoqual article	+0 dB	1 minute	+0 dB	1 minute

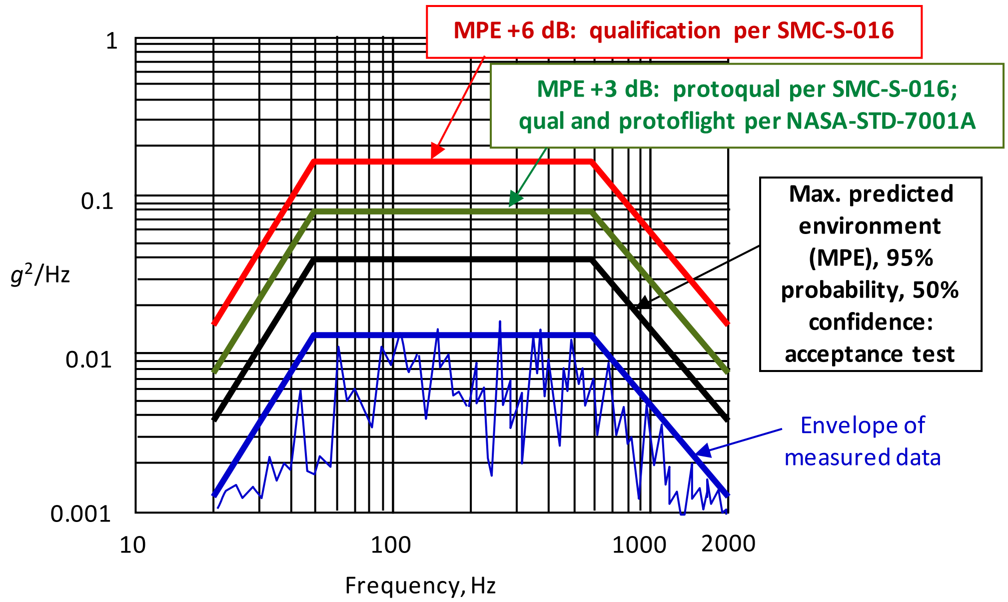


Fig. 5-10. How Test Levels Relate to the Maximum Predicted Environment. ASD levels uniformly double with each 3 dB increase. (Using MPE for acceptance testing, as shown here, is based on the assumption that MPE exceeds any applicable minimum workmanship levels. If not, the acceptance test is to be done to the envelope of MPE and minimum workmanship levels, and the 3 dB and 6 dB margins for the other tests shown here apply to that envelope.)

The SMC-S-016 criteria apply to small (less than about 400 lb) satellites only. The DOD traditionally does not put larger spacecraft on shakers. Instead, the structure for such spacecraft goes through static loads testing, most components and subsystems are tested for random vibration, and the integrated vehicle is tested for acoustics. NASA's philosophy for large spacecraft is quite different: static loads testing and component-level random vibration testing, followed by vehicle-level acoustic testing and vibration testing on a shaker; the latter may be a random vibration test at low frequency or a sine-vibe test.

Testing a small spacecraft for random vibration on a shaker produces component responses that are more flight-like than is achieved during component-level testing. Component-level testing is typically performed to a prescribed ASD on a rigid fixture, with no simulation of the frequency-dependent ***impedance*** (resistance to interface acceleration) of the actual flight mounting structure, and thus tends to be more severe.[6] Ideally, component-level test environments are derived to envelop the expected responses during vehicle-level testing. Some amount of over-testing at the component level is desirable because we want little chance of failure during the vehicle-level test.

Establishing Test Environments at Spacecraft and Component Levels

Deriving or identifying appropriate test environments can be difficult, both at the spacecraft level and at the component level. For each rideshare (multiple payloads) mission, there is usually something unique about the configuration of the payload stack. This means there probably won't be measured flight data

[6] The same is true when testing a small spacecraft on a shaker, which is why we notch or force limit the test. See Part 6 in this series.

that directly apply to your small satellite's random vibration environment. You'll be testing components for random vibration prior to the test of the integrated spacecraft, which means you won't know what levels to use for the component tests.

Many programs deal with uncertainty in spacecraft- and component-level environments by using the generalized component-level environments from GEVS (Ref. 5), Table 2.4-3. Figure 5-11 shows this table. According to our sources at NASA Goddard Space Flight Center, these environments were derived to envelop available flight data from multiple launch vehicles based on component mass.

Because a small satellite on a rideshare mission is analogous to a component in a large satellite, the mass-appropriate GEVS environment is sometimes used for testing of small satellites in absence of better information. The highest of the GEVS levels, which applies to components weighing up to 50 lb, is often used for testing of small-satellite components. Unfortunately, this approach can lead to problems.

As an example, let's say we plan to do a protoflight test on a 400-lb satellite to the lowest qualification levels shown in Fig. 5-11. Prior to that, we test a 3-lb reaction wheel to the highest (50-lb component) qualification levels. Then, when we test the satellite, we measure responses at the base of the reaction wheel. Even though the RMS response may be below the 14.1 *g*-rms component test levels, within certain frequency ranges the response is most likely well higher than those test levels as a result of the spacecraft structure's modal response. If the wheel has one or more natural frequencies within those ranges, there may be considerable risk that it will fail during the spacecraft test, even though it passed the component-level test.

To avoid such situations, a good approach is to do a vibration test of the spacecraft structure in advance of the flight-component tests, with mass simulators representing the flight components. (See Part 2 of this series for guidance on design of mass simulators.) The test article can be either the flight structure or a dedicated structure for qualification testing. During test, measure component responses, following the guidance in Table 2-1 from Part 2 of this series. After the test, draw a smooth-line envelope over the measured responses at qualification levels, and then use those envelopes for component testing.

If the spacecraft structure will not be tested prior to component testing, we recommend that you subject a detailed finite element model (FEM) of the spacecraft to the specified random vibration environment, and predict response ASDs at component mounting locations. Then, for component testing, use the envelope of the predicted responses and the GEVS environment for 50-lb components. In any case, try to select robust components that have been qualification tested to levels that are well higher than the GEVS qualification levels for 50-lb components.

Frequency (Hz)	ASD Level (g^2/Hz)	
	Qualification	Acceptance
20	0.026	0.013
20-50	+6 dB/oct	+6 dB/oct
50-800	0.16	0.08
800-2000	-6 dB/oct	-6 dB/oct
2000	0.026	0.013
Overall	14.1 G_{rms}	10.0 G_{rms}

The acceleration spectral density level may be reduced for components weighing more than 22.7-kg (50 lb) according to:

	Weight in kg	Weight in lb	
dB reduction	= 10 log(W/22.7)	10 log(W/50)	
$ASD_{(50\text{-}800\ Hz)}$	= 0.16•(22.7/W)	0.16•(50/W)	for protoflight
$ASD_{(50\text{-}800\ Hz)}$	= 0.08•(22.7/W)	0.08•(50/W)	for acceptance

Where W = component weight.

The slopes shall be maintained at + and - 6dB/oct for components weighing up to 59-kg (130-lb). Above that weight, the slopes shall be adjusted to maintain an ASD level of 0.01 g^2/Hz at 20 and 2000 Hz.

For components weighing over 182-kg (400-lb), the test specification will be maintained at the level for 182-kg (400 pounds).

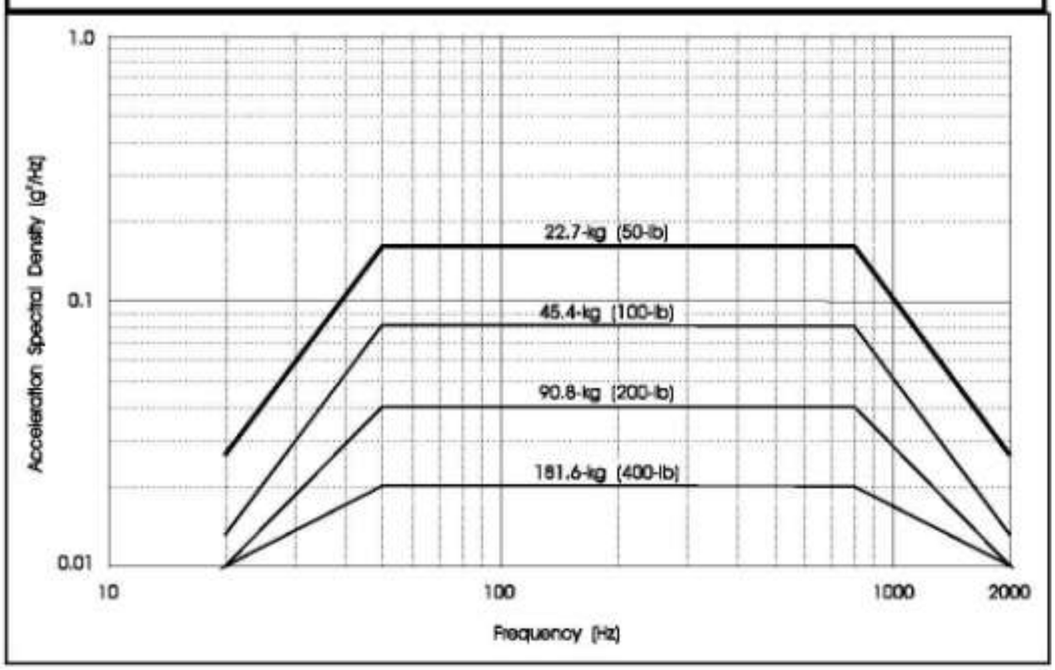


Fig. 5-11. GEVS Generalized Random Vibration Test Levels for Components Weighing Up to 400 lb. (Ref. 5) Although commonly used in the small-satellite industry, the levels shown here for 50-lb components may not be appropriate for small-satellite components, as noted in the text.

Test-Control Tolerances

Table 5-2 lists the acceptable tolerances on how well the shaker must control the environment to a specified test environment. The table also shows constraints on frequency bandwidth for data resolution, which is discussed below.

Table 5-2. NASA and Air Force Standards for Test-Control Tolerances and Frequency Bandwidth. (Ref. 1, sec. 4.3.4.1, and Ref. 4, Sec. 4.7) Tolerance on ASD applies at any given frequency within the applicable range; tolerance on g-rms (bottom row) ensures that the ASD doesn't fall too much above or below the specified levels overall.

Frequency Range	Test Parameter	NASA-STD-7001A	SMC-S-016
20 - 100 Hz	Acceleration spectral density	± 3 dB	± 1.5 dB
	Maximum bandwidth for data resolution	25 Hz	10 Hz
100 - 1000 Hz	Acceleration spectral density	± 3 dB	± 1.5 dB
	Maximum bandwidth for data resolution	25 Hz	10% of mid-band freq.
1000 - 2000 Hz	Acceleration spectral density	± 3 dB	± 3 dB
	Maximum bandwidth for data resolution	25 Hz	100 Hz
20 - 2000 Hz	Overall level, g-rms	± 10%	± 1 dB (± 12%)

Data Resolution

Resolution of the data processing affects the accuracy of a controlled environment and also of data interpretation. Table 5-2 shows government standards for maximum permissible bandwidth for data resolution, but these values are outdated; today's digital controllers can do much better—and should. There are good reasons for using higher resolution.

The higher the resolution, the more accurately the test environment is controlled to the specification and the more accurately we'll be able to reduce the response data to useful information. On the other hand, the argument against high resolution is that it takes more time for the controller to make the necessary calculations and adjust input as needed to stay within the specified tolerances. Additional time in test means more unwanted fatigue damage to the test article.

The best resolution is the highest that still allows the control system to perform responsively. This resolution depends on the control system. As noted above, newer digital controllers can handle more lines of control. 2.5 Hz resolution for a test from 20 to 2000 Hz (792 lines) is easily achievable with relatively new controllers, and you'll probably be able to do better.

Figure 5-12 shows a hypothetical example of what can happen with resolution that is too low.

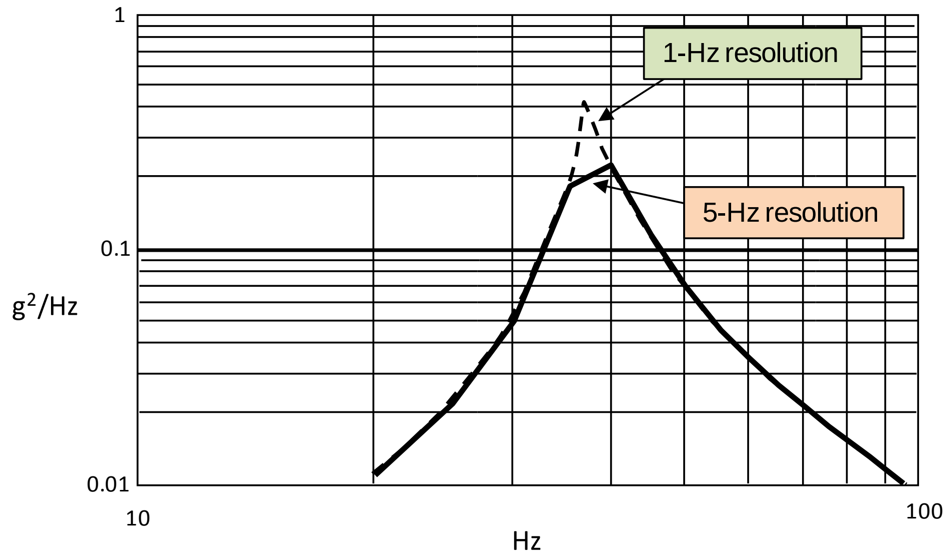


Fig. 5-12. Hypothetical Example of Response Truncation from Low Data Resolution. The RMS for this response would be significantly under-calculated if a 5-Hz bandwidth is used. The error is potentially even greater when we consider that, with 5-Hz bandwidth in this case, the input environment is not accurately controlled at low frequencies[7]. For example, even though the input ASD is at the specified level over the full frequency band of, say, 35 – 40 Hz, the input at 36 Hz may be higher than specified, and the input at 38 Hz may be lower.

Interpreting Test Data

Let's look now at some plots of response data and see what they tell us. Figure 5-13 shows a comparison of data from a random vibration test and a low-level sine sweep for the same accelerometer. Overall, these two response plots look much the same, with peaks indicating modes of vibration. The primary response is at about 230 Hz in each plot. The vertical scale for a sine sweep is *g*, whereas the scale for random vibration is g^2/Hz.

For a linear system, at a given frequency, we expect the ratio of response to input in the sine sweep (if the sweep rate is low enough) to be equal to the square root of the response ratio from the random vibration plot. This is not the case for the plots in Fig. 5-13. Input levels, shown as dashed lines, are 0.03 g^2/Hz for the random test and 0.1 *g* for the sine sweep. For the response peak, 20/0.1 = 200 for the sine sweep vs. $\sqrt{400/0.03}$ = 115 for random vibration. This comparison implies (we would have to know the data resolution and sine-sweep rate to know for sure) that damping is higher (lower Q) for the random vibration test, which loaded the structure much more severely than the sine sweep. (In both tests, the dynamic amplification was quite high!) Damping typically increases with load level, as preloaded surfaces slip more (e.g., bolt threads relative to nut or insert threads). Friction at slipping surfaces is usually the biggest contributor to damping.

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[7] As indicated by the transmissibility curve (see Part 1 of this series, Fig. 1-2), the actual bandwidth for data processing is not important to accuracy; it's the ratio of that bandwidth to the test article's natural frequency of interest. Thus, a constant bandwidth affects accuracy more at low response frequencies. If the control system has the capability, it makes more sense accuracy-wise to use a graduated scale for bandwidth—say, 5% of band-center frequency—than a constant bandwidth.

The comparison in Fig. 5-13 also shows a slight difference in frequency between tests for the primary response peak. By eye, it appears that the frequency of the peak in the random vibration test is a few percent lower that it is in the sine sweep. This is another indication of the nonlinearity typical in a structure with fastened joints. Natural frequency often drops somewhat at higher levels of excitation.

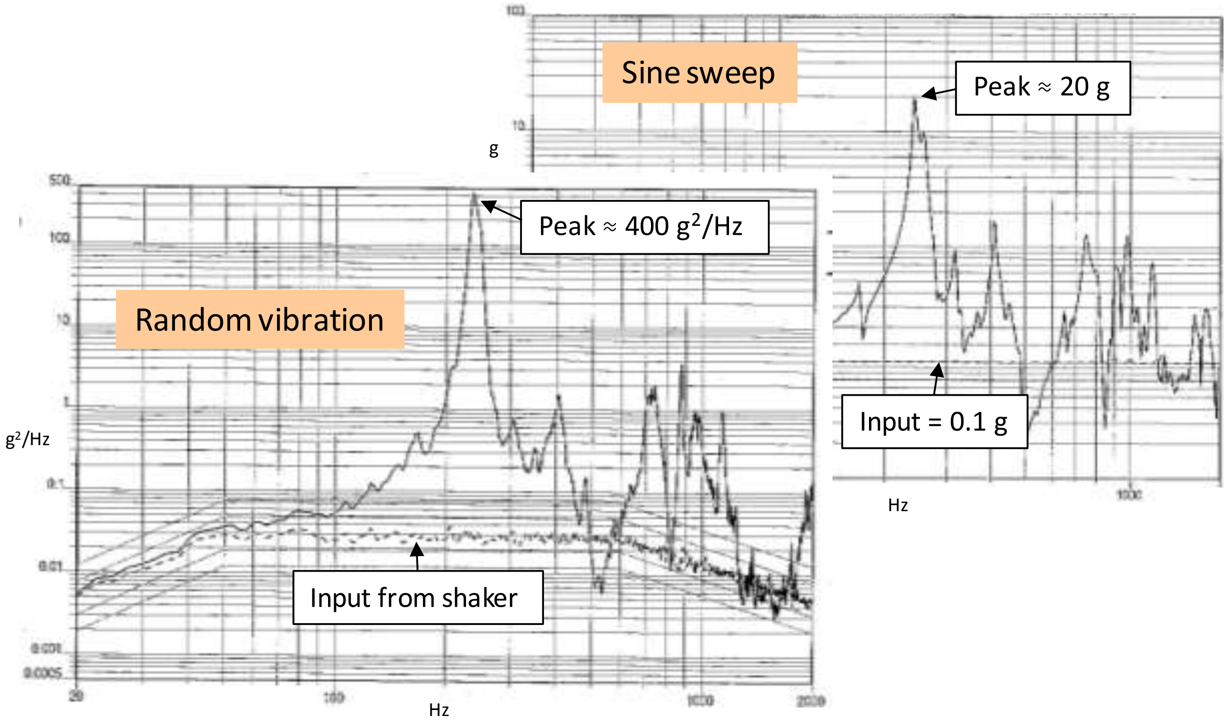

Fig. 5-13. Example of Random Vibration Response Data Compared with Response to Low-level Sine Sweep at the Same Accelerometer Location. Note the similarity in the plots.

In a random vibration test, we often want to know the peak total force or moment at the base of the test article (***base load***). The most direct way to determine these loads is to sandwich force gages between the test article and the shaker. (See Part 6 of this series.) Alternatively, in absence of such gages, we can use a finite element model (FEM) in combination with test data to estimate the base loads.

In most cases, the first lateral mode of a satellite cantilevered from a shaker (or the launch vehicle) is a rocking mode in which most of the strain energy is at the base, divided between the separation mechanism and the satellite structure local to the separation mechanism. In such a mode, most of the satellite's mass acts like a rigid body rotating on an angular spring. As a result, if we measure acceleration at the top and bottom of the test article, we can derive the CG acceleration and the angular acceleration, and then use them to estimate force and moment at the base from the mass properties of the rocking rigid body. A FEM can help us understand how much of the mass is actually moving.

Example 5-2: Estimating Base Moment from Response Data

Consider the response plot in Fig. 5-14. The accelerometer was mounted on the top of a small satellite during a lateral test. The big peak at 182 Hz is for response of the spacecraft's rocking mode. The total response at this location up to 2000 Hz is 54 g-rms. Let's say we want to estimate the base moment from this data.

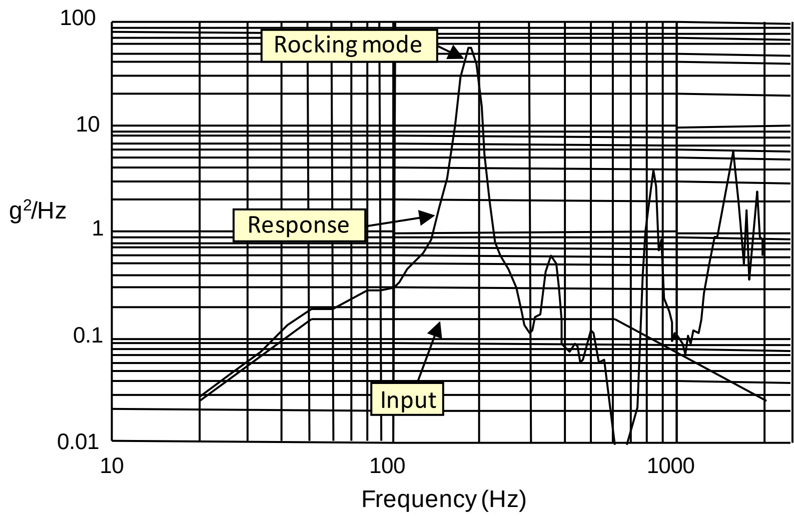

Fig. 5-14. Example Problem: Lateral Acceleration at the Top of a Small Satellite.

Recall that the RMS is very nearly equal to the square root of the area under the ASD. For this example, the square root of the area under the entire response plot, from 20 – 2000 Hz, is 54 g. Recall also that the RMS value is the standard deviation of the random acceleration. However, not all of the 54 g-rms value is associated with the rocking mode. The peaks above 300 Hz indicate additional modes of vibration. Let's assume only the rocking mode contributes significantly to base moment for this small spacecraft[8]. Accordingly, let's consider the RMS only to 300 Hz when estimating base moment (shaded region in Fig. 5-15). Numerical integration tells us the RMS from 20 to 300 Hz is about 41 g. This, then, is the approximate standard deviation of acceleration at the top, from 20 to 300 Hz, with most of the acceleration associated with the rocking mode.

[8] To see if this is the case for your spacecraft, calculate modal effective mass (see part 1 of this series of papers) with a FEM. For many spacecraft, the rocking mode has over 90% of the modal effective mass for the associated rotational degree of freedom, which means only the rocking mode will contribute significantly to base moment.

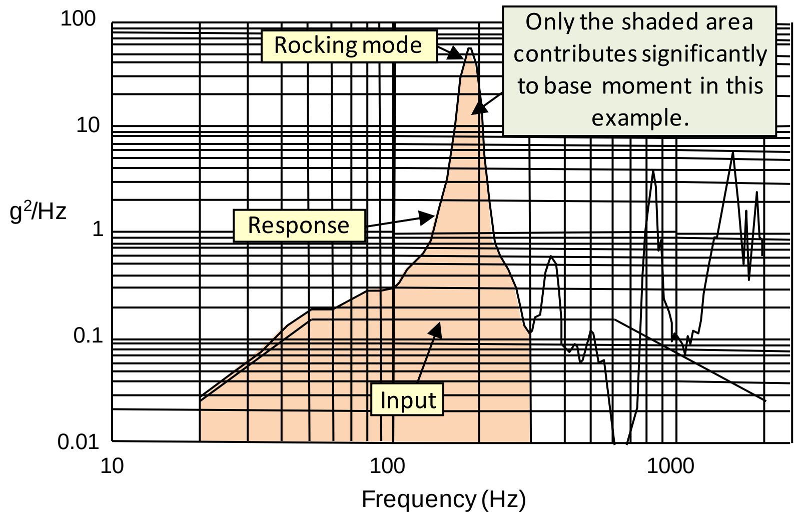

Fig. 5-15. Example Problem: Region of the Response ASD Associated with the Rocking Mode.

Next, we try to understand the mode shape. Figure 5-16 shows the test configuration and the approximate shape of the rocking mode, as calculated with a FEM and substantiated by test measurements at multiple locations. Note that, for this example, only the outer box structure is rocking; the inner column has very little modal displacement. At low-frequency input, the column should move nearly as a rigid body with the shaker, contributing little to the base moment.

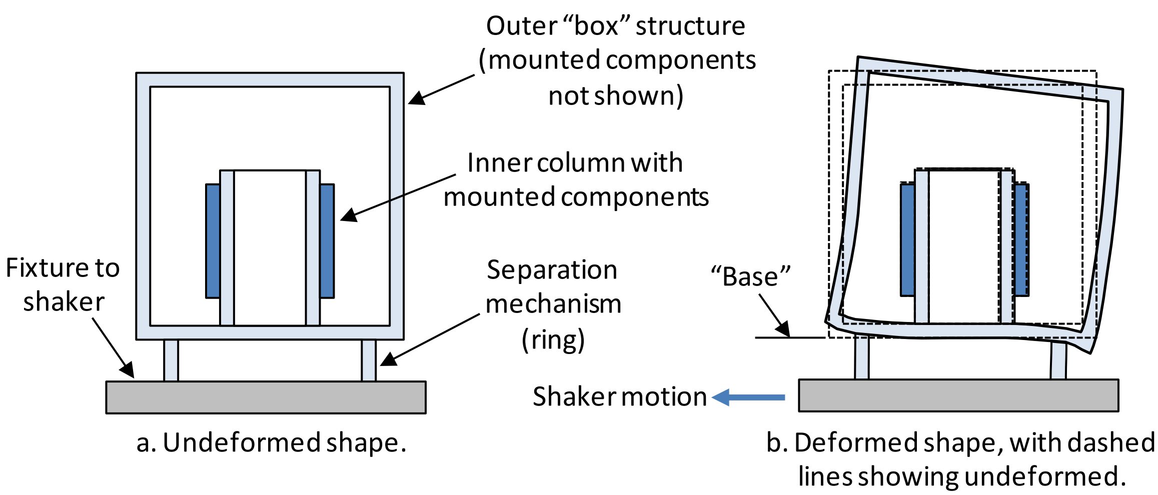

Fig. 5-16. Cut-away (Internal) View of the Satellite that Produced the Response Plots in Figs. 5-14 and 5-15. The outer structure is a hollow cubic box. We want to estimate moment at the base, which is defined here as the interface between the satellite and the separation mechanism.

Because the inner column isn't moving much, we'll ignore its contribution to base moment. The outer box structure is a 12.5" hollow cube that weighs about 25 lb, including mounted components. Thus, its mass, *m*, is 25 lb/(386 in/s^2) = 0.0648 lb-s^2/in. The box's mass moment of inertia, *J*, (at its CG and about the axis out of the plane of the drawing in Fig. 5-16) is 2.50 lb-s^2-in.

With these mass properties in hand, we'll be able to estimate base moment if we know the box's average translational acceleration, as calculated at the CG, and angular acceleration. To estimate these rigid-body accelerations from test data, we'll need to know the dimensions shown in Fig. 5-17.

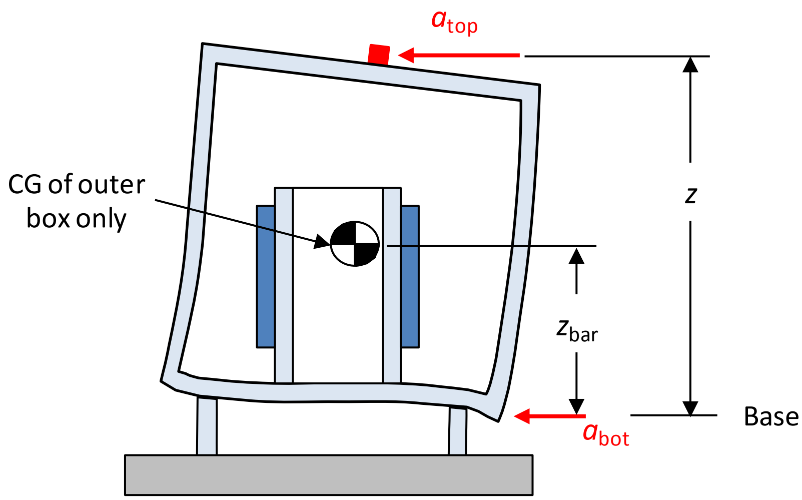

Fig. 5-17. Dimensions Needed to Estimate Translational and Angular Acceleration of the Box. a_{top} and a_{bot} are the measured accelerations at top and bottom, respectively. Because we are ignoring the base moment caused by the mass of the inner column, as discussed in the text, we also ignore it when calculating the CG.

In this example, z_{bar} = 6.25" and z = 12.5". As explained above, we've estimated a_{top} to be 41 *g*-rms. Let's say we don't have a direct measurement of a_{bot}, so we'll assume the box bottom is moving with the shaker, without amplification. The square root of the area under the input ASD from 20 – 300 Hz is about 6 *g*-rms; this is what we'll use for a_{bot}. The RMS CG accelerations are

$$a_{cg} = a_{bot} + (a_{top} - a_{bot})\left(\frac{z_{bar}}{z}\right)$$

$$= 6 + (41-6)\left(\frac{6.25}{12.5}\right) = 23.5\, g\text{ - rms}$$

$$R = (a_{top} - a_{bot})(g)\left(\frac{1}{z}\right)$$

$$= (41-6)(386\text{ in/s}^2)\left(\frac{1}{12.5\text{ in}}\right) = 1080\text{ rad/s}^2\text{, rms}$$

where a_{cg} is the translational acceleration at the CG and R is the angular acceleration. The above equations are based on the assumptions that (a) the top and bottom accelerations are caused solely by rocking superimposed with rigid-body translation, and (b) the box rocks as a rigid body (not the case here, based on Fig. 5-17, but the assumption is not too far off).

Ignoring the contribution of the inner column, we estimate RMS base moment, M, as

$$
\begin{aligned}
M &\approx a_{cg} w z_{bar} + RJ \\
&= 23.5(25\,\text{lb})(6.25\,\text{in}) + (1080\,\text{rad/s}^2)(2.50\,\text{lb}\cdot\text{s}^2\cdot\text{in}) \\
&= 3670 + 2700 = 6370\,\text{in}\cdot\text{lb, rms}
\end{aligned}
$$

The 3-sigma base moment is then estimated at 3(6370) = 19,100 in-lb.

There are several things to note in this example:

- We were diligent with units to make sure they balance.
- When g is the unit of acceleration, we use the acceleration (number of g's) as a unit-less ***load factor***, which is a multiplier for weight (in units of force, such as lb), in order to calculate ***inertia force***, which is the force that resists acceleration. For example, an object weighing 10 lb under 5 g acceleration from its mounting structure exerts an inertia force of -50 lb on the mounting structure Acceleration and inertia load, hence load factor, have opposite signs—if one is positive, the other is negative—but that distinction is not important in random vibration because acceleration oscillates from positive to negative.
- We would have significantly underestimated the base moment if we had ignored angular acceleration, which caused 42% of the total.

If we want to improve our estimate, we can add the contribution of the inner column mass, which we ignored above. We'd assume the column mass is accelerating as a rigid body with the shaker at 6 g-rms from 20 – 300 Hz. If the column weighs 5 lb and its CG is 5" above the base, the extra RMS moment from the column is approximately 6 x 5 x 5 = 150 in-lb. This isn't much when compared with the RMS moment of 6370 in-lb that we estimated when ignoring the column. There will be a higher frequency mode in which the column rocks, with dynamic amplification, but associated contribution to base moment would be more difficult to approximate with simple methods. When more accuracy is needed, we should use a FEM to predict the RMS base loads.

Let's look at some more random vibration test data (Fig. 5-18). What does this data tell you? Give it some thought before reading further.

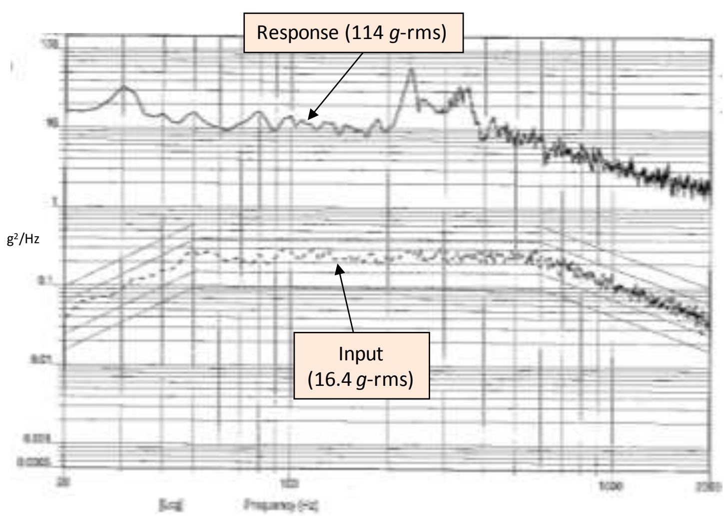

Fig. 5-18. Example of Response in the Same Axis as the Input (Test Axis). What does this response plot tell you? Hint: Remember the transmissibility curve. (See Part 1 of this series, Fig. 1-2).

There's something clearly wrong with the data in Fig. 5-18. No system can respond as the plot indicates; at every frequency, response is about two orders of magnitude higher than the input. The transmissibility curve tells us that, for a sinusoidally base-driven mass on a spring, response at low frequency is nearly the same as the input. As the frequency of excitation increases, response builds up, peaking at resonance, then declines, becoming less than the input (isolation) at high frequencies. Thus, with multiple modes of vibration responding to random vibration across a broad spectrum, we expect that response will be higher than input at some frequencies and lower than input at others. And, at frequencies well below the first response peak, we expect response to track input nearly one for one. Figure 5-18 tells us two things:

1. Most likely, the calibration factor[9] is wrong for the accelerometer.
2. Because response doesn't ramp up between 20 – 50 Hz, as does the input, the accelerometer is not measuring accurately within that frequency range.

The lesson here is to make sure you do sanity checks on the test data—and do so before tearing down the test configuration! Afterwards, you won't be able to investigate and correct the problem. You'll be forced to either make assumptions regarding the data or discard it completely.

Part 6 of this series addresses notching and force limiting.

References

1. NASA-STD-7001A. "Payload Vibroacoustic Test Criteria." National Aeronautics and Space Administration. January 20, 2011.
2. Lang, George, and John Van Baren. "How Well Does 3 Approximate ∞? Understanding +/-3σ Clipping in Random Shake Tests." Vibration Research Corporation. 2007.
3. Sarafin, Thomas P., ed. *Spacecraft Structures and Mechanisms: From Concept to Launch*. Microcosm Press. 1995.
4. SMC-S-016 (adapted from MIL-STD-1540, which is no longer active). "Test Requirements for Launch, Upper Stage and Space Vehicles." Air Force Space Command. September 5, 2014.
5. GEVS: GSFC-STD-7000A. "General Environmental Verification Standard for GSFC Flight Programs and Projects." NASA Goddard Space Flight Center. April 22, 2013.

[9] A ***calibration factor*** for an instrument is the factor that relates the parameter of interest, such as acceleration, to voltage (or whatever other parameter is being measured directly).

